Super-Kamiokande as a probe of CP Violation

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We show that if the presently observed L/E-flatness of the electron-like event ratio in the Super-Kamiokande atmospheric neutrino data is confirmed then the indicated ratio must be *unity*. Further, it is found that once CP is violated the exact L/E flatness implies: (a) The CP-violating phase, in the standard parameterization, is narrowed down to two possibilities $\pm\pi/2$, and (b) The mixing between the second and the third generations must be maximal. With these results at hand, we argue that a dedicated study of the L/E-flatness of the electron-like event ratio by Super-Kamiokande can serve as an initial investigatory probe of CP violation in the neutrino sector.

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The Super-Kamiokande data on the atmospheric neutrinos have opened a new realm of physics research [1]. The simplest interpretation of these data is flavor oscillations arising from neutrino being linear superposition of some underlying mass eigenstates. This circumstance not only takes us into the physics beyond the standard model of the high energy physics, but it also allows to probe various aspects of quantum gravity [2]. As such, much theoretical and experimental effort is being devoted to deciphering the nature of neutrino. Here, using a very specific aspect of the Super Kamiokande data, we shall analytically constrain the CP-violating neutrino oscillation mixing matrix. This would help the design of future experiments, allow for more analytically-oriented theoretical research, and provide a new direction of research at the existing experimental facilities.

This *Letter* joins the on-going research with the observation that as soon as the first results from the Super-Kamiokande on atmospheric neutrinos became available, one of us emphasized that the L/E flatness noted in the *abstract* places a set of constraints on the neutrino oscillation mixing matrix [3]. However, in that, and our subsequent work [4], CP violation has been neglected. Apart from reasons of simplicity, there is no *a priori* reason to assume the absence of CP violation in the neutrino sector. In addition, the observed cosmological baryonic asymmetry may be deeply connected with a CP violation in the leptonic sector [5]. This becomes particularly important, as we shall comment below, if the neutrino-sector CP violation is affected by gravity. As such, we present here a non-trivial generalization of the constraints presented in the early work [3,4] to obtain a CP-violating bimaximal matrix for neutrino oscillations [6].

To generalize the discussion of Refs. [3,4], we start from the probability formula of neutrino oscillations. As in the quark sector, when neutrinos have non-zero masses, their weak eigenstates may not coincide with the mass eigenstates, but may be linear superposition of the mass eigenstates. The latter choice is precisely what is suggested by the existing data [1,7–9]. As such, in a phenomenology of neutrino oscillations, a flavor eigenstate of a neutrino is postulated to be a linear superposition of some underlying mass eigenstates

$$\mid \nu_\alpha\rangle = \sum_j U_{\alpha j} \mid \nu_j\rangle, \tag{1}$$

where $U_{\alpha j}$ is an element of the mixing matrix with $\alpha = e,\ \mu,$ or τ and $j = 1,\ 2,\ 3$ in the framework of three generations. In the literature, U is usually taken as the standard parameterization matrix [10]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{2}$$

multiplied by a phase matrix

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3+\delta_{13}} \end{pmatrix} \tag{3}$$

if neutrinos are of the Majorana type. Here, $c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}$, and ϕ_2 and ϕ_3 are the additional phases for Majorana neutrinos. Due to the unobservable effect of P in flavor oscillation experiments, we shall drop it in the discussion and simply equate the mixing matrix U to V in calculations that follow. Furthermore, θ_{12}, θ_{23}, and θ_{13} in U can all be made to lie in the first quadrant by an appropriate re-definition of the relevant fields.

Assuming the underlying mass eigenstates to be relativistic in the observer's frame [11], the flavor-oscillation probability is given by [4,12]

$$P(\nu_\alpha \xrightarrow{L} \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})\sin^2(\varphi_{jk}) + 2\sum_{j<k} \mathrm{Im}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})\sin(2\,\varphi_{jk})\,. \quad (4)$$

where L (measured in meters) refers to the source-detector distance, and the flavor-oscillation inducing kinematic phases φ_{ij}, are defined as

$$\varphi_{ij} = 1.27\,\Delta m^2_{ij}\frac{L}{E}, \quad (5)$$

where E (measured in MeV) refers to the "energy" of the flavor state, and $\Delta m^2_{ij} = m^2_i - m^2_j$ is the mass-squared difference of the underlying mass eigenstates (measured in eV^2).

For the CP conjugate channel, the CP-odd term, that is, the last term in Eq. (4), changes sign:

$$P(\bar\nu_\alpha \xrightarrow{L} \bar\nu_\beta) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})\sin^2(\varphi_{jk}) - 2\sum_{j<k} \mathrm{Im}(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k})\sin(2\,\varphi_{jk})\,. \quad (6)$$

Note that, all $\mathrm{Im}(U_{\alpha j}U.\beta j^* U^*_{\alpha k}U_{\beta k})$ with $\alpha \neq \beta$ and $j \neq k$ take the same value $J_{CP} = c_{12}s_{12}c_{23}s_{23}c^2_{13}s_{13}s_\delta$ ($s_\delta = \sin\delta_{13}$, $c_\delta = \cos\delta_{13}$), which is the measure of CP violation [13].

The Super-Kamiokande measured ratio $\mathcal{R}_e$ of the electron-like events is defined as

$$\mathcal{R}_e = \frac{N'_e + N'_{\bar e}}{N_e + N_{\bar e}}. \quad (7)$$

where N_e and $N_{\bar e}$ are the numbers of *predicted* ν_e and $\bar\nu_e$ events in the absence of neutrino oscillations, whereas the primed quantities are the corresponding numbers of *observed* events, allowing for the presence of neutrino oscillations.

If at the top of atmosphere, i.e. at the "source," the number of ν_e ($\bar\nu_e$) and ν_μ ($\bar\nu_\mu$) are $N_{\nu_e}(N_{\bar\nu_e})$ and $N_{\nu_\mu}(N_{\bar\nu_\mu})$ respectively, with the cross-sections for ν_e and $\bar\nu_e$ are σ_{ν_e} and $\sigma_{\bar\nu_e}$, we obtain the following set of event predictions at the detector:

$$N_e = N_{\nu_e}\sigma_{\nu_e} \quad (8)$$

$$N_{\bar e} = N_{\bar\nu_e}\sigma_{\bar\nu_e} \quad (9)$$

$$N'_e = N_{\nu_e}P(\nu_e \xrightarrow{L} \nu_e)\sigma_{\nu_e} + N_{\nu_\mu}P(\nu_\mu \xrightarrow{L} \nu_e)\sigma_{\nu_e} \quad (10)$$

$$N'_{\bar e} = N_{\bar\nu_e}P(\bar\nu_e \xrightarrow{L} \bar\nu_e)\sigma_{\bar\nu_e} + N_{\bar\nu_\mu}P(\bar\nu_\mu \xrightarrow{L} \bar\nu_e)\sigma_{\bar\nu_e}. \quad (11)$$

The first two equations correspond to the absence of flavor oscillations, while the last two equations incorporate the effects of flavor oscillations of neutrinos. Inserting Eqs. (8-11) into Eq. (7), and taking note of the fact that due to CPT symmetry,

$$P(\nu_e \xrightarrow{L} \nu_e) = P(\bar\nu_e \xrightarrow{L} \bar\nu_e),$$

we arrive at

$$\mathcal{R}_e - P(\nu_e \xrightarrow{L} \nu_e) = \frac{N_{\nu_\mu}P(\nu_\mu \xrightarrow{L} \nu_e)\sigma_{\nu_e} + N_{\bar\nu_\mu}P(\bar\nu_\mu \xrightarrow{L} \bar\nu_e)\sigma_{\bar\nu_e}}{N_{\nu_e}\sigma_{\nu_e} + N_{\bar\nu_e}\sigma_{\bar\nu_e}}. \quad (12)$$

Finally, on defining

$$\frac{N_{\bar{\nu}_e}}{N_{\nu_e}} = x, \qquad \frac{N_{\bar{\nu}_\mu}}{N_{\nu_\mu}} = y$$
$$\frac{\sigma_{\bar{\nu}_e}}{\sigma_{\nu_e}} = \lambda, \qquad \frac{N_{\nu_\mu}}{N_{\nu_e}} = r, \tag{13}$$

it is easy to show that

$$\mathcal{R}_e - P(\nu_e \xrightarrow{L} \nu_e) = \frac{r}{1+\lambda x}\left(P(\nu_\mu \xrightarrow{L} \nu_e) + \lambda y P(\bar{\nu}_\mu \xrightarrow{L} \bar{\nu}_e)\right). \tag{14}$$

Now, substituting Eqs. (4,6) into the above equation, and after simplifying, we obtain

$$\begin{aligned}
&\left\{|U_{e1}|^2\,|U_{e2}|^2 + r\frac{1+\lambda y}{1+\lambda x}\mathrm{Re}(U_{\mu 1}U_{e1}^* U_{\mu 2}^* U_{e2})\right\}\sin^2(\varphi_{12}) \\
&+ \left\{|U_{e1}|^2\,|U_{e3}|^2 + r\frac{1+\lambda y}{1+\lambda x}\mathrm{Re}(U_{\mu 1}U_{e1}^* U_{\mu 3}^* U_{e3})\right\}\sin^2(\varphi_{13}) \\
&+ \left\{|U_{e2}|^2\,|U_{e3}|^2 + r\frac{1+\lambda y}{1+\lambda x}\mathrm{Re}(U_{\mu 2}U_{e2}^* U_{\mu 3}^* U_{e3})\right\}\sin^2(\varphi_{23}) \\
&- \frac{r}{2}\frac{1-\lambda y}{1+\lambda x}J_{CP}\left[\sin(2\,\varphi_{12}) + \sin(2\,\varphi_{13}) + \sin(2\,\varphi_{23})\right] \\
&= \frac{1}{4}\left(1 - \mathcal{R}_e\right).
\end{aligned} \tag{15}$$

It is worth noting that in the case $x = y$ and $J_{CP} = 0$, i.e., if the ratio of the numbers of $\bar{\nu}_e$ to ν_e equals the ratio of the numbers of $\bar{\nu}_\mu$ to ν_μ at the source , and if there is no CP violation in the neutrino sector, Eq. (15) looses any dependence on the neutrino and anti-neutrino cross sections, σ_{ν_e} and $\sigma_{\bar{\nu}_e}$.

In order for Eq. (15) to hold for all values of L/E we must impose the constraints:

$$\frac{r}{2}\,\frac{1-\lambda y}{1+\lambda x}\,J_{CP} = 0 \tag{16}$$

and

$$|U_{e1}|^2\,|U_{e2}|^2 + r\,\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 1}U_{e1}^* U_{\mu 2}^* U_{e2}) = 0 \tag{17}$$

$$|U_{e1}|^2\,|U_{e3}|^2 + r\,\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 1}U_{e1}^* U_{\mu 3}^* U_{e3}) = 0 \tag{18}$$

$$|U_{e2}|^2\,|U_{e3}|^2 + r\,\frac{1+\lambda y}{1+\lambda x}\,\mathrm{Re}(U_{\mu 2}U_{e2}^* U_{\mu 3}^* U_{e3}) = 0. \tag{19}$$

Since Eq. (15) holds for any value of L/E, we are also free to set $L/E = 0$, which yields:

$$\mathcal{R}_e = 1. \tag{20}$$

Although we invoke the Super-Kamiokande observed flatness for $\mathcal{R}_e$ from the beginning, we did *not* refer to a specific value of $\mathcal{R}_e$. The present analysis *predicts* $\mathcal{R}_e$ to be unity. This circumstance is in sharp contrast to the framework of references [3,4] where one assumes both the indicated flatness and the value unity for $\mathcal{R}_e$.

Furthermore, Eq. (16) requires that $J_{CP} = 0$ and/or $\lambda y = 1$. The case $J_{CP} = 0$ has been discussed extensively in Refs. [3,4]. Here we take $J_{CP} \neq 0$, and hence study $\lambda y = 1$. According to the definition, $\lambda y = 1$ indicates that, if the ratio of the numbers of ν_μ to $\bar{\nu}_\mu$ is close to the ratio of the cross-sections of $\bar{\nu}_e$ to ν_e, then this circumstance allows to ignore the last term on the left hand side of Eq. (15). From Table 1 of Ref. [14] we estimate $y \approx 2.06 \pm 0.31$,[1] while from Ref. [15] we infer $\lambda \approx 1/2.4$. Thus, the required condition is fulfilled on "accidental" grounds. Further justification for ignoring the indicated term lies in the fact that J_{CP} is significantly suppressed by data-indicated $U_{e3} \ll 1$.

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[1]It being the value associated with the lowest atmospheric density in the experiment, identified here as "the top of the atmosphere."

Substituting the relevant elements of U into Eqs. (17-19), we obtain

$$c_{12}s_{12}c_{13}^2 + \frac{2r}{1+\lambda x}\{c_{12}s_{12}(s_{23}^2 s_{13}^2 - c_{23}^2) + (s_{12}^2 - c_{12}^2)c_{23}s_{23}s_{13}c_\delta\} = 0 \tag{21}$$

and

$$c_{12}s_{13} - \frac{2r}{1+\lambda x}s_{23}(c_{12}s_{23}s_{13} + s_{12}c_{23}c_\delta) = 0 \tag{22}$$

$$s_{12}s_{13} - \frac{2r}{1+\lambda x}s_{23}(s_{12}s_{23}s_{13} - c_{12}c_{23}c_\delta) = 0. \tag{23}$$

From Eqs. (22,23) we infer that

$$s_{23}^2 = \frac{1+\lambda x}{2r}, \tag{24}$$

and

$$c_\delta = 0, \tag{25}$$

which implies that the CP phase is $\pi/2$ or $-\pi/2$.

Inserting Eqs. (24,25) into Eq. (21) yields

$$c_{23}^2 = \frac{1+\lambda x}{2r}. \tag{26}$$

Finally, combining Eq. (24) and Eq. (26), we obtain:

$$\theta_{23} = \pi/4, \qquad r = 1+\lambda x, \tag{27}$$

That is, the mixing between the second and the third generations is maximal, and that the ratio of the numbers of ν_μ to ν_e equals to one plus the ratio of the numbers of $\bar{\nu}_e$ to ν_e events in case of no oscillations.

As a result, the indicated L/E flatness in the the Super-Kamiokande data on the atmospheric neutrinos implies CP-violating maximal mixing matrix:

$$U^\pm = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & \mp i\,s_{13} \\ -\frac{1}{\sqrt{2}}\,(s_{12} \pm i\,c_{12}\,s_{13}) & \frac{1}{\sqrt{2}}\,(c_{12} \mp i\,s_{12}\,s_{13}) & \frac{1}{\sqrt{2}}c_{13} \\ \frac{1}{\sqrt{2}}\,(s_{12} \mp i\,c_{12}\,s_{13}) & -\frac{1}{\sqrt{2}}\,(c_{12} \pm i\,s_{12}\,s_{13}) & \frac{1}{\sqrt{2}}c_{13} \end{pmatrix} \tag{28}$$

where U^+ corresponds to $\delta_{13} = \pi/2$, and U^- arises from $\delta_{13} = -\pi/2$. Corresponding to these two general forms for U, we obtain the following two measures of CP violation:

$$J_{CP}^\pm = \pm\frac{1}{2}c_{12}s_{12}c_{13}^2 s_{13} = \pm\frac{1}{8}\sin(2\theta_{12})\sin(2\theta_{13})\cos(\theta_{13}) \tag{29}$$

In the limit θ_{13} vanishes the $U^\pm$ reduces to the result contained in Eq. (26) of Ref. [4]. Preliminary indications that the U matrix carries the general form given in Eq. (28) can also be deciphered from a recent work of Barger, Geer, Raja, and Whisnant [16]. Furthermore, for $\theta_{12} = \pi/4$, U^+ coincides with the Xing postulate [17].

Since the CHOOZ experiment [18] constraints, for large-δm^2, $\sin^2(2\theta_{13})$ to be about 0.1, even the large value of $\delta_{13} = \pm\pi/2$ implied by the present analysis, does not result in a maximal CP-violating difference:

$$P(\nu_\alpha \xrightarrow{L} \nu_\beta) - P(\bar{\nu}_\alpha \xrightarrow{L} \bar{\nu}_\beta) = 4J_{CP}\sum_{j<k}\sin(2\,\varphi_{jk}) \tag{30}$$

However, we note that Eqs. (4,6) define a set of flavor-oscillation clocks, and these clocks must red-shift when introduced in a gravitational environment. If this environment is characterized by a dimensionless gravitational potential, Φ_{grav}, then in order that the flavor-oscillations suffer a gravitationally-induced red-shift we must replace, in Eq. (30), φ_{jk} by $(1+\Phi_{grav})\,\varphi_{jk}$. For other quantum-gravity effects on neutrino oscillations we refer the reader to Ref. [15]. Such gravitationally-induced modifications to a neutrino-sector CP violation may carry significant physical implications.

In summary, we find that if CP is violated in neutrino sector, the exact L/E flatness of $\mathcal{R}_e$ implies that: (i) The mixing between the second and the third generations must be maximal, (ii) The ratio $\mathcal{R}_e$ must be unity, (iii) The CP-violating phase in the standard parameterization matrix is $\pi/2$ up to a sign ambiguity, (iv) $N_{\nu_\mu}\sigma_{\nu_e} = N_{\bar{\nu}_\mu}\sigma_{\bar{\nu}_e}$, and finally that (v) $N_{\nu_\mu}/N_{\nu_e} = 1 + N_{\bar{\nu}_e}\sigma_{\bar{\nu}_e}/N_{\nu_e}\sigma_{\nu_e}$. Therefore, a dedicated study of the ratio $\mathcal{R}_e$ in terms of its precise value, and its L/E dependence, can become a powerful probe to study CP violation in the neutrino sector. Within the framework of this *Letter*, if the future data confirms $\mathcal{R}_e$ to be unity for all zenith angles, then we must conclude that either there is no CP violation in the neutrino sector, or it is of the form predicted by equation (29). This precise result, in conjunction with knowledge of θ_{12}, θ_{13}, and the associated mass-squared differences, up to a sign ambiguity, completely determines the expectations for CP violation in all neutrino-oscillation channels.

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